Kirchoff's Current Law

So, we now know that:

$$I = \frac{dQ}{dt} = \iint_S \mathbf{J}(\bar{r}) \cdot \overline{ds}$$

Consider now the case where S is a **closed** surface:

$$I = \frac{dQ}{dt} = \oiint_S \mathbf{J}(\bar{r}) \cdot \overline{ds}$$

The current I thus describes the rate at which **net** charge is **leaving** some **volume V** that is surrounded by surface S.

We will find that **often** this rate is I=**0** !

Q: *Yikes! Why would this value be zero??*

A: Because charge can be neither **created** nor **destroyed**!

Think about it.

If there was some **endless** flow of charge crossing closed surface S—**exiting** volume V—then there would have to be some "fountain" of charge creating this endless **outward** flow.

Alternatively, if there was some **endless** flow of charge crossing closed surface S—**entering** volume V—then there would have to be some charge "drain" that disposed of this endless **inward** flow.

* But, we **cannot** create or destroy charge—**endless** charge fountains or charge drains **cannot** exist!

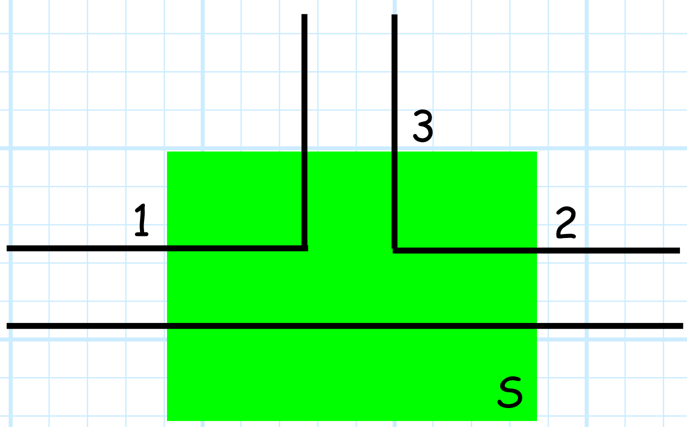
* Instead, charge **exiting** volume V through surface S must have likewise **entered** volume V through surface S (and vice versa).

* As a result, the rate of net charge flow (i.e., **current**) across a **closed** surface is very often **zero**!

In other "words", we can state:

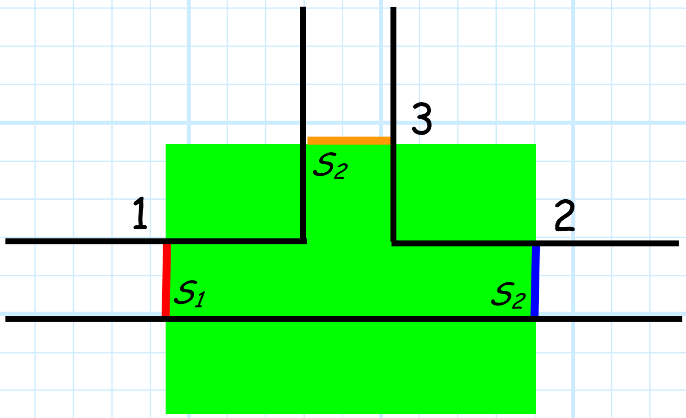
$$\oiint_S \mathbf{J}(\bar{r}) \cdot \overline{ds} = 0$$

For example, consider a closed surface S that surrounds a "node" at which 3 conducting **wires** converge:



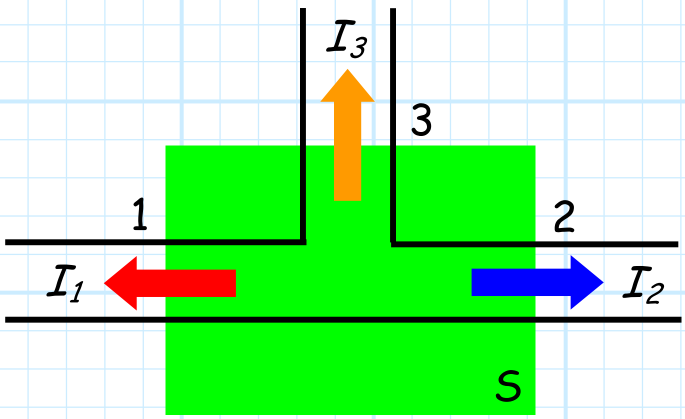
Since current is flowing **only** in these wires, the surface integral reduces to a surface integration over the cross section of **each** of the three wires:

$$\oint\!\!\oint_S \mathbf{J}(\bar{r}_s)\cdot\overline{ds} = \oint\!\!\oint_{S_1} \mathbf{J}(\bar{r}_s)\cdot\overline{ds_1} + \oint\!\!\oint_{S_2} \mathbf{J}(\bar{r}_s)\cdot\overline{ds_2} + \oint\!\!\oint_{S_3} \mathbf{J}(\bar{r}_s)\cdot\overline{ds_3}$$



The result of each integration is simply the **current** flowing in each wire!

$$\oint\!\!\oint_S \mathbf{J}(\bar{r}_s)\cdot\overline{ds} = I_1 + I_2 + I_3$$



But remember, since we know that charge cannot be created or destroyed, we have concluded that:

$$\oiint_S \mathbf{J}(\bar{r}_s) \cdot \overline{ds} = 0$$

Meaning:

$$0 = I_1 + I_2 + I_3$$

More generally, if this node had n wires, we could state that:

$$0 = \sum_n I_n$$

Hopefully you recognize this statement—it's **Kirchoff's Current Law**!

Therefore, a more general, **electromagnetic** expression of Kirchoff's Current Law is:

$$\oiint_S \mathbf{J}(\bar{r}) \cdot \overline{ds} = 0$$

Note that this result means that the current density $\mathbf{J}(\bar{r})$ (for this case) is **solenoidal**!

In other words, the above integral **likewise** means that $\nabla \cdot \mathbf{J}(\bar{r}) = 0$.

Gustav Robert Kirchhoff (1824-1887), German physicist, announced the laws that allow calculation of the currents, voltages, and resistances of electrical networks in 1845, when he was only **twenty-one** (so what have **you** been doing)! His other work established the technique of spectrum analysis that he applied to determine the composition of the Sun.